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A NUMERICAL ANALYSIS OF MHD CONVECTIVE HEAT TRANSFER THROUGH A HORIZONTAL WAVY CHANNEL WITH RADIATION EFFECT

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ABSTRACT

The Radiation effect on the unsteady MHD convection flow through a Non-uniform horizontal channel. The unsteadiness is due to the imposed oscillatory flux on the convection flow through the non-uniform channel. The perturbation analysis is carried out with the slope of the boundary as the perturbation parameter. The velocity and temperature profiles were plotted and their behavior is discussed in detail. The Stress and the average Nusselt number are also calculated and tabulated for these sets of parameters.

KEYWORDS: Heat transfer, Oscillatory flux, radiation effect and Wavy channel.

INTRODUCTION

Unsteady convection flows play an important role in aerospace technology, turbo-machinery and chemical Engineering. Such flows arise due to either unsteady motion of boundary or boundary temperature. Unsteadiness may also be due to oscillatory free stream velocity or temperature. These oscillatory free convective flows are important from technological point of view.

The heat transfer through wavy channels has been a topic of interest in recent times owing to its applications in technological areas viz., transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching of an ablative surface and film vaporization in combustion chambers. The MHD effects on the oscillating convection flows have been discussed by several authors [1,2,3].

This problem has been extended to the case of wavy walls by [4]. The rate of thermal radiation is a major important in some industrial application such a glass production and furnace design and space technology applications. Such as cosmical flight aerodynamics rocket, propulsion systems, plasma physics and space craft reentry aero thermodynamics which operate at high temperatures. [5] derived exact solution to fully developed vertical channel for a radiative effect on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method. [6] Considered flow of an electrically conducting and heat generating fluid over an isothermal wedge. [7] studied the effects of radiation and mass transfer effects on a flow past a moving vertical cylinder using Rosseland approximation by the Crank-Nicolson finite difference method. [8] analyzed the effects of radiation and mass transfer on flow past a semi-infinite vertical plate.

In this paper, we investigate the effect or radiation on unsteady convective heat transfer flow of a viscous fluid through a horizontal wavy channel. By using a regular perturbation method, the non-linear coupled equations of flow and heat transfer are solved. The effect of radiation on flow and heat transfer characteristics is discussed in detail.

FORMULATION OF THE PROBLEM

We consider the unsteady motion of a viscous, incompressible electrically conducting fluid through a porous medium in a horizontal channel bounded by wavy walls in the presence of a constant heat source /sink. A uniform

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magnetic field of strength 'Ho' is applied normal to the walls. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy and Ohmic dissipations are neglected in comparison to the flow by conduction and convection. Also, the kinematic viscosityν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system O (x, y) with x-axis in the direction of motion and y-axis in the vertical direction and the walls are taken at y = ± Lf(δx/L), where 2L is the distance between the walls, f is a twice differentiable function and δ is a small parameter proportional to the boundary slope. A linear density temperature variation is assumed with ρ_e and T_e is the density and temperature in the equilibrium state. The magnetic Reynolds number R_m is chosen to be much less than unity so that the induced magnetic field can be neglected in comparison to the applied field. The flow is maintained by an oscillatory volume flux rate for which a characteristic velocity is defined as

$$q(1+ke^{i\varpi t})=(\frac{1}{L})\int_{-Lf}^{Lf}udy \qquad (1)$$

The equations governing the unsteady magneto hydrodynamic flow and heat transfer in Cartesian coordinate system O(x,y,z), in the absence of any input electric field are

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2)$$

Equation of linear momentum

$$\rho_e(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-(\frac{\sigma\mu_e^2H_o^2}{\rho_o})u \qquad (3)$$

$$\rho_e(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\rho g \qquad (4)$$

Equation of energy

$$\rho_eC_p(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y})=\lambda(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+Q-\frac{\partial(q_r)}{\partial y} \qquad (5)$$

Equation of state

$$\rho-\rho_e=-\beta\rho_e(T-T_e) \qquad (6)$$

In the equilibrium state

$$0=-\frac{\partial p_e}{\partial x}-\rho_e g \qquad (7)$$

Where $p=p_e+p_D, p_D$ being the hydrodynamic pressure and in this state the temperature, gradient balances the heat flux generated by source Q.

The flow being two-dimensional, in order to maintain the compatability of the equations **(4)** and **(6)** the temperature in the flow field must be a general function of x and y. However, temperature over the boundary walls may be kept constant assuming the temperature dependence on the walls to be a function of $\eta = y / f(x)$.

The boundary conditions for the velocity and temperature fields are

$$u=0,\ v=0, T=T_1(\eta) \text{ On } y = -L f(\delta x/L)$$

$$u=0,\ v=0, T=T_2(\eta) \text{ On } y = L f(\delta x/L) \quad (8)$$

Invoking Rosseland approximation (Brewester) for the radiative flux we get

$$q_r = \frac{4\sigma}{3\beta_R}\frac{\partial(T'^4)}{\partial y} \quad (9)$$

Expanding T'^4 in Taylor series about T_e and neglecting higher order terms

$$T'^4 \approx 4TT_e^3 - 3T_e^4 \quad (10)$$

In view of the continuity equation **(3)** we define the stream function ψ as

$$u = \frac{\partial\psi}{\partial y} \quad , \quad v = -\frac{\partial\psi}{\partial x} \quad (11)$$

Eliminating pressure p from equations **(3)** & **(4)** and using **(11)** the equations Governing the flow in terms of ψ are

$$[(\nabla^2\psi)_t + \psi_x(\nabla^2\psi)_y - \psi_y(\nabla^2\psi)_x] = \nu\nabla^4\psi + \beta g(T - T_0)_x - (\frac{\sigma\mu_e^2 H_o^2}{\rho_e})\frac{\partial^2\psi}{\partial y^2} \quad (12)$$

$$\rho_e C_p(\frac{\partial T}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}) = k_1\nabla^2\theta + Q + \frac{16T_e^3\sigma^*}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \quad (13)$$

Introducing the non-dimensional variables in **(12)** & (**13)** as

$$x' = x/L,\ y' = y/L, t' = t\varpi, \Psi' = \psi/qL, \theta = \frac{T - T_e}{T_2 - T_e} \quad (14)$$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$\gamma^2((\nabla^2\psi)_t + R\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}) = \nabla^4\psi + (\frac{G}{R})\theta_x - M^2\frac{\partial^2\psi}{\partial y^2} \quad (15)$$

and the energy equation in the non-dimensional form is

$$P_1\gamma^2(\frac{\partial\theta}{\partial t}) + P_1 R(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}) = \nabla^2\theta + \alpha_1 \quad (16)$$

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With the corresponding boundary conditions

$$\psi(+1)-\psi(-1)=(1+ke^{i\varpi t})$$

$$\frac{\partial\psi}{\partial x}=0,\ \frac{\partial\psi}{\partial y}=0\ ,\theta=\frac{T_1-T_e}{T_2-T_e}=h, say\quad on\ y=-f(\delta x)$$
$$\frac{\partial\psi}{\partial x}=0,\ \frac{\partial\psi}{\partial y}=0\ ,\theta=1\ \ on\ y=f(\delta x)$$
$$\frac{\partial\theta}{\partial y}=0,\quad at\quad y=0 \tag{17}$$

Where

$$R=\frac{qL}{\nu}$$ (The Reynolds number)

$$G=\frac{\beta g(T_2-T_e)L^3}{\nu^2}$$ (The Grashof number)

$$\mathrm{P}=\frac{\mu C_p}{k_1}$$ (The Prandtl number)

$$M^2=(\frac{\sigma\mu_e^2H_o^2L^2}{\nu^2})$$ (The Hartman number)

$$\gamma^2=\left(\frac{\varpi L^2}{\nu}\right)$$ (The wormeley number)

$$\alpha=\frac{QL^2}{k}$$ (The Heat source parameter)

$$N=\frac{4\sigma^* T_e^3}{\beta_R k_1}$$ (The radiation parameter)

$$P_1=\frac{3NP}{3N+4}\ ,\qquad \alpha_1=\frac{3N\alpha}{3N+4}$$

SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the channel walls is given by

$$\tau=\frac{\sigma_{xy}(1-f'^2)+(\sigma_{yy}-\sigma_{xx})f'^2}{(1+f'^2)}$$

Where

$$\sigma_{ij}=-p\delta_{ij}+2\mu e_{ij}$$

$$\sigma_{xx}=\frac{\partial u}{\partial x},\ \sigma_{yy}=\frac{\partial v}{\partial y},\ \sigma_{zz}=\frac{\partial w}{\partial z},\sigma_{xy}=0.5(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})$$

And the corresponding expressions are

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$$(\tau)_{\eta=+1} = ((1-f'^2)f^2(e_{48}+ke^{it}e_{50})+\delta((1-f'^2)f^2(g_{21}+ke^{it}g_{10})-(\frac{2f'}{f})(g_{25}+ke^{it}g_{26})+O(\delta^2))/(1+f'^2)$$

$$(\tau)_{\eta=-1} = ((1-f'^2)f^2(e_{49}+ke^{it}e_{50})+\delta((1-f'^2)f^2(g_{22}+ke^{it}g_{20})-(\frac{2f'}{f})(g_{29}+ke^{it}g_{31})+O(\delta^2))/(1+f'^2)$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{f(\theta_m-\theta_w)}(\frac{\partial\theta}{\partial y})_{\eta=\pm1}$$

Where

$$\theta_m = 0.5\int_{-1}^{1}\theta\,dy$$

And the corresponding expressions are

$$(Nu)_{\eta=+1} = \frac{2(e_{40}+\delta(e_{41}+ke^{it}e_{42})}{f(e_3+\delta(e_{38}+ke^{it}e_{39})-1)}$$

$$(Nu)_{\eta=-1} = \frac{2(e_{43}+\delta(e_{44}+ke^{it}e_{45})}{f(e_{37}+\delta(e_{38}+ke^{it}e_{39})-h)}$$

Where $e_1, e_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, e_{40}, \ldots. e_{50}, g_1, g_2, \ldots.., g_{31}$ are constants

DISCUSSIONS AND CONCLUSIONS OF THE NUMERICAL RESULTS

The primary aim of our analysis is to investigate the radiation effect on the behavior of the temperature induced buoyancy force taking in to account the effect of surface geometry and wall temperature ratio. The flow is analyzed for different sets of the parameters β and N_1. It should be noted that the flow is basically asymmetric due to distinct surface temperatures. For computation purpose, we assume the boundaries to be $y = \pm f(\bar{x}) = \pm(1+\beta e^{-x^{-2}})$ and β>0 corresponds to dilated channel and β<0 corresponds to constricted channel. The transformation $\eta = \frac{y}{f(\bar{x})}$ reduce the boundaries to η=±1. We confine our attention to dilated channel.

The influence of surface geometry on the flow phenomena is exhibited in Fig.1 The reversal flow which appears in entire flow region for β=0.3, disappears for higher β≤0.6 and reappears in the fluid region, for β=0.9. Higher the dilation of the channel walls larger the magnitude of u. With reference to the radiation parameter N_1 we notice that the axial velocity 'u' enhances with increase in N_1 (Fig. 2). The secondary velocity 'v' which arises due to the non-uniformity of the boundary is exhibited in Figs. 3&4. The variation of v with β shows that greater the dilation larger the magnitude of v (Fig.3). The variation of 'v' with radiation parameter N_1 shows that |v| enhances marginally with increase in N_1. (Fig.4).

The influence of the amplitude β of the boundary curve is exhibited in Fig.5. Higher the dilation of the channel walls larger the temperature in the flow region except in a narrow region abutting the lower boundary η= -1 where θ depreciates. From Fig.6. We find that the temperature in flow field enhances everywhere with increase in the radiation parameter N_1 except in a narrow region adjacent to η=-1, where it experiences depreciation.

The Shear stress at the boundaries η=±1 are evaluated for different variations of G, and N_1 are presented in Tables. 1 – 2. The effect of the radiation parameter N_1 on τ is to enhance its magnitude for all G fixing the other parameters (Tables.1&2).

The Nusselt number (Nu) which measures the rate of heat transfer at the boundaries has been numerically evaluated for different values of the governing parameters. It is found that the Nusselt number is positive at both the walls for all variations. From Tables. 3 & 4. we find that at η=1 the rate of heat transfer reduces with $N_1 \leq 5$ and for higher $N_1>5$ we find an enhancement in 'Nu', while at η=-1, |Nu| experiences an enhancement with N_1 for all G.

Table.1. Shear Stress (τ) at η=1 P=0.71, R=35, M=2, α=2, β=0.5, x=π/4, t=π/4

G/τ	I	II	III	IV	V	VI
$5x10^2$	-0.01639	-0.016396	-0.069158	-0.073169	-0.082607	-0.093224
10^3	-1.288326	-1.328776	-1.394509	-1.402581	-1.421576	-1.442944
$3x10^3$	-1.634558	-1.715752	-1.847693	-1.863897	-1.902021	-1.944913
$-5x10^2$	4.714257	4.734859	4.768339	4.772450	4.782125	4.793008
-10^3	8.167914	8.208868	8.275419	8.283593	8.302822	8.324455
$-3x10^3$	17.246970	17.328670	17.461430	17.477730	17.516100	17.559260

Table.2. Shear Stress (τ) at η=-1P=0.71, R=35, M=2, α=2, β=0.5, x=π/4, t=π/4

G/τ	I	II	III	IV	V	VI
$5x10^2$	0.001384	0.001386	0.054145	0.058156	0.067594	0.078212
10^3	1.258299	1.298749	1.364482	1.372555	1.391549	1.412917
$3x10^3$	1.574496	1.655690	1.787631	1.803835	1.841959	1.884851
$-5x10^2$	-4.699246	-4.719849	-4.753328	-4.757441	-4.767115	-4.77799
-10^3	-8.137896	-8.178849	-8.245401	-8.253574	-8.272803	-8.294437
$-3x10^3$	-17.18695	-17.26865	-17.40140	-17.41770	-17.45607	-17.49923

N_1	0.4	1.0	4.0	5.0	10.0	100.0

Table.3. Nusselt number (Nu) at η=1P=0.71, R=35, M=2, α=2, β=0.5, x=π/4, t=π/4

G/Nu	I	II	III	IV	V	VI
$5x10^2$	-0.9902	-0.9906	0.1215	0.1732	0.2844	0.3944
10^3	-1.0157	-0.4486	0.1261	0.1787	0.2917	0.4032
$3x10^3$	-1.0919	-0.4744	0.1354	0.1904	0.3082	0.4240
$-5x10^2$	-0.9596	-0.4327	0.1123	0.1628	0.2715	0.3793
-10^3	-0.9535	-0.4326	0.1076	0.1577	0.2657	0.3728
$-3x10^3$	-0.9581	-0.4398	0.0978	0.1476	0.2550	0.3616

Table.4. Nusselt number (Nu) at η=-1 P=0.71, R=35, M=2, α=2, β=0.5, x=π/4, t=π/4

G/Nu	I	II	III	IV	V	VI
$5x10^2$	-1.477889	-1.477899	-1.630559	-1.638507	-1.655879	-1.673464
10^3	-1.504284	-1.571119	-1.646782	-1.654159	-1.670269	-1.686556
$3x10^3$	-1.581166	-1.635469	-1.696029	-1.701882	-1.714632	-1.727478
$-5x10^2$	-1.445685	-1.523782	-1.613276	-1.622064	-1.641295	-1.660793
-10^3	-1.439055	-1.519499	-1.611840	-1.620917	-1.640785	-1.660937
$-3x10^3$	-1.443843	-1.527269	-1.623041	-1.632455	-1.653064	-1.643968

N_1	0.4	1.0	4.0	5.0	10.0	100.0

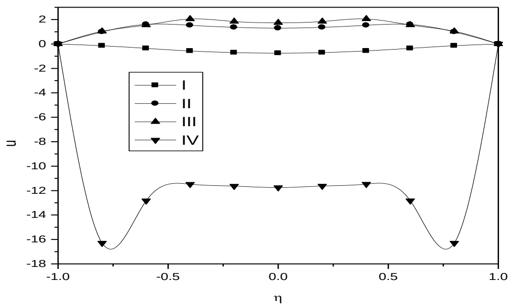

Fig .1. Variation of u with β

	I	II	III	IV
N_1	4	5	10	100

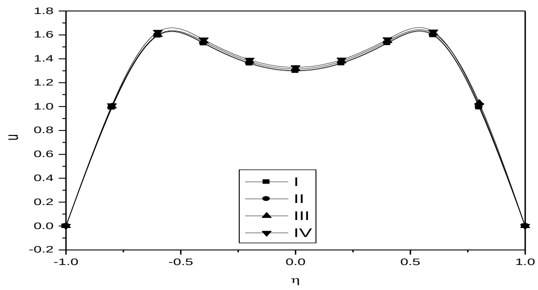

Fig .2. Variation of u with N_1

	I	II	III	IV
β	0.3	0.5	0.7	0.9

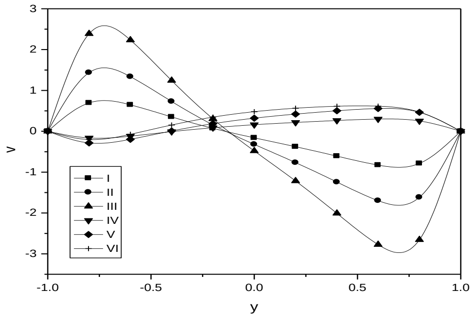

Fig.3. Variation of v with β

$G=5x10^2$, R=35, M=2, α=2, β=0.5, x=π/4, t=π/4

	I	II	III	IV
N_1	4	5	10	100

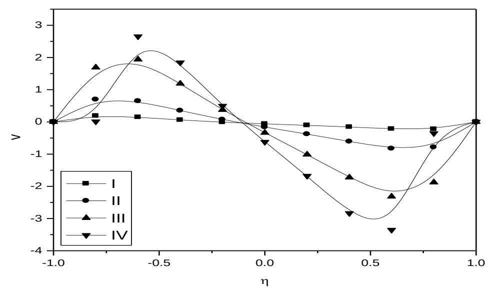

Fig.4. Variation of v with N_1

$G=5x10^2$, R=35, M=2, α=2, $N_1=4$, x=π/4, t=π/4

	I	II	III	IV
β	0.3	0.5	0.7	0.9

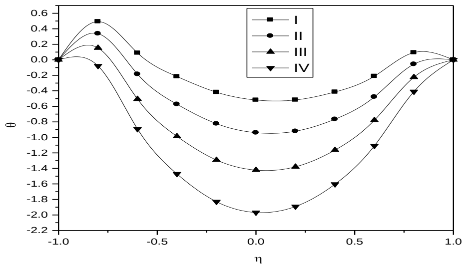

Fig.5. Variation of θ with β

$G=5x10^2$, R=35, M=2, α=2, $N_1=4$, x=π/4, t=π/4

	I	II	III	IV
β	0.3	0.5	0.7	0.9

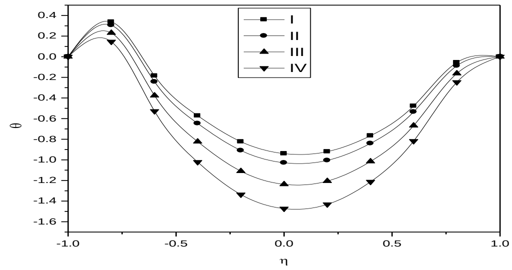

Fig.6. Variation of θ with N_1

$G=5x10^2$, R=35, M=2, α=2, β=0.5, x=π/4, t=π/4

	I	II	III	IV
N_1	4	5	10	100

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NOMENCLATURE

u Velocity (m/s)
p Pressure(N/m^2)
T Temperature in the flow region (K)
T_e Temperature in equilibrium state (K)
Q Strength of the heat source (W)
C_p Specific heat at constant pressure(J/kg K)
q_r Radiative heat flux (W/m^2)

Greek Symbols

ρ_e Te density of the fluid in the equilibrium state (Kg/m^3)
ρ Density of the fluid (Kg/m^3)
μ_e Te magnetic permeability(H/m)
μ Coefficient of viscosity (Ns/m^2)
λ Coefficient of thermal conductivity (W/mk)
β Coefficient of thermal expansion(m^3)
σ Electrical conductivity (ms^3A^2/kg)
δ Porosity of the medium

REFERENCES

[1] Brewester, M. (1992): Thermal radiative transfer and properties, John Wiley & Sons. Inc., New York.
[2] Ganeshan,P.and Loganathan,P.(2002): Radiation and Mass Transfer effects on flow of an Incompressibleviscous fluid past a moving cylinder, Int.J.Heat Mass Transfer, Vol.45, pp.4251- 4288.
[3] Chamkha, A.J. (2000): Thermal radiation and buoyancy effects on hydromagnetic flow over An accelerating permeable surface with heat or sink, Int.J.Heat Mass Transfer, Vol.38, pp.1699-1712.
[4] Desikachar, K.S.and Ramachandra Rao, A. (1985): Effect of Magnetic field on the flow and Blood oxygenation in channels of variable cross section, Int.J.Eng.Sci.Vol.23, pp.1121-1133.
[5] Chamkha, A.J.Mujtaba, M.Suadri,A and Issac,(2003): Thermal radiation effects on MHD Forced convection flow adjacent to a non-isothermal wedge in the presence of heat source or Sink, Int.J.Heat Mass Transfer, Vol.39, pp.305-312.
[6] Raptis, A. (2002): Radiation and free convection flow through porous cavity using thermal Non-equilibrium model, Int.J.Therm.Sci.Vol.41, pp.861-870.
[7] Grief,R.Habib,I.S. and Lin L.C.(1971):Laminar convection of radiating gas in a vertical Channel, J.FluidMech.Vol.45, pp.513-520.
[8] Raptis,A. (1998): Radiation and free convection flow through a porous medium, Int.Commn.Heat Mass Transfer. Vol. 25, pp.289-295.